St Paul’s First School Numeracy Update
Concrete vs abstract

• When we were children, maths consisted of learning all the important steps and rules. We were not given manipulatives, nor an explanation for the rules, or the meanings behind the maths. We did fractions “this way” because that was the way to do it, multiplication tables were memorised because the teacher said to memorise them. It was just that way. The “old school” way.

• The current thinking is that we move away from concrete apparatus too quickly, expecting our children to use abstract thinking in Key Stage 1, before they are ready for it.
Manipulatives

- Manipulatives are defined as concrete objects (things you can touch and move around) that aid in classification, patterning, counting, equations, fractions, multiplication, and other maths tasks.

- They include blocks, geometric shapes, base ten blocks (Dienes), unifix cubes, fraction bars, and plastic counting cubes. They may also include everyday items such as buttons, bingo tokens, beans, dice, straws, craft sticks, food items such as jelly beans, and marbles.
Some questions to consider

• Would using manipulatives more often in the classroom make a difference to how the children learn maths?
• Are manipulatives really so great and wonderful, or are they being overused?
• Do they prevent children from developing abstract thinking skills, memorising and learning multiplication facts, recalling facts quickly, restrict their imagination, decrease their ability to solve everyday problems and critical thinking?
• Have you ever been to a shop and the cashier was unable to count out how much change to give you back?
What does 28 look like?
25

20 + 5 = 25
5 + 20 = 25
10 + 15 = 25
50 - 25 = 25
60 - 35 = 25
20 - 45 = 25

25
20 + 5 = 25
5 + 10 = 25
10 + 10 = 20
5 + 5 = 25
Teaching Column Addition and Subtraction

Using place value counters, Dienes blocks, money
Addition

• Column addition is straightforward when there is no exchanging involved.
• E.g. 44 + 32 = 76
Addition

The language of column addition is “exchanging” not “carrying” as most of us will remember from school.

- We exchange units for tens, tens for hundreds, etc.

- There are several resources that can be used to support the teaching of column addition that involves exchanging: Dienes blocks, place value counters and coins.
Column addition involving exchanging
Dienes blocks TU + TU

47 + 26 =
47 + 26 =

By putting the post-it note at the side, this counts as a written method.

This could be photographed as assessment evidence.
47 + 26 =

Combine the tens and units together in the bottom section of the grid.
47 + 26 =

If we look at the units first,
7 + 6 = 13.

Can we write 13 in the units column?
47 + 26 =

Demonstrate the process of exchanging ten of the units for a ten.

Have one child as a banker. The children can take their units to be exchanged.
47 + 26 =

Now exchange the ten into the tens column.

Ask the children to count how many tens are in the tens column and how many units are in the units column.
47 + 26 = 73
Place value counters HTU + HTU

345 + 226 =

<table>
<thead>
<tr>
<th>Thousands</th>
<th>Hundreds</th>
<th>Tens</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100 100</td>
<td>10 10</td>
<td>1 1</td>
</tr>
<tr>
<td>100 100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>10 10</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
345 + 226 =

<table>
<thead>
<tr>
<th>Thousands</th>
<th>Hundreds</th>
<th>Tens</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>100</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>100</td>
<td>10</td>
<td>1</td>
</tr>
</tbody>
</table>
345 + 226 =

First add the units: $5 + 6 = 11$

Talk to the children about whether we can write 11 in one column. Why not? What do we need to do instead?
345 + 226 =

We have taken 10 of the units and swapped them for a ten. We have now added the ten into the tens column of the place value chart.
We can now add up tens and the hundreds to find the answer.

\[ 345 + 226 = 571 \]
Money

\[
\begin{array}{|c|c|c|}
\hline
\text{£} & \text{10p} & \text{1p} \\
\hline
\text{£2.47} & \text{2 x 10p} & \text{16 x 1p} \\
\hline
\text{£3.55} & \text{3 x 10p} & \text{30 x 1p} \\
\hline
\end{array}
\]

\[£3.55 + £2.47 = \]
First add the pennies: $5p + 7p = 12p$

Discuss how we can’t write 12p in this column.
Exchange ten 1ps for a 10p coin.
Add this into the 10p column.
Now add the 10ps.

There are ten 10ps.

Discuss what to do now.
Exchange the ten 10ps for a £1 coin and add this into the £ column.
Now find the total:

£3.55 + £2.47 = £6.02
Subtraction

• Column subtraction is straightforward when there is no exchanging involved.
• E.g. $54 - 22 = 32$
Subtraction

• The main error that children make with column subtraction is when deciding whether the calculation is possible without exchanging:
  - E.g. 234
    \[\underline{116}\]
  The children will look at this and think “I can’t do 4-6 so I’ll do 6-4 instead”.
Dienes blocks TU-TU

33 – 16 =

Discuss that we can’t do the calculation:

3 – 6

So what can we do?
33 – 16 =

Exchange a ten for ten units.
Now we have

13 – 6 = 7

We can now do this calculation.

Again have a banker on the table who the children can go to when they are swapping.
33 – 16 =

Now we can do the subtraction in the tens column:

20 – 10 = 10
$33 - 16 = 17$
Place value counters HTU - HTU

632 – 214 =

Discuss how we can’t do 2 – 4 =
632 – 214 =

Now we have 12 – 4.
We can do this calculation.

$$12 - 4 = 8$$
Now we need to look at the tens column.

$$20 - 10 =$$
20 – 10 = 10
Finally we need to look at the hundreds column.

600 – 200 = 400
<table>
<thead>
<tr>
<th>Thousands</th>
<th>Hundreds</th>
<th>Tens</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100 100</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>100 100</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

632 – 214 = 418
Compacted

<table>
<thead>
<tr>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>

or

leading to

<table>
<thead>
<tr>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>

\[
\begin{array}{c}
2 \ 5 \\
+ \ 4 \ 7 \\
\hline
\end{array}
\]
Compacted or leading to
Money

<table>
<thead>
<tr>
<th>£</th>
<th>10p</th>
<th>1p</th>
</tr>
</thead>
<tbody>
<tr>
<td>£1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>10p</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1p</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

£4.23 - £2.35 =
We can’t do 3p – 5p so we will need to exchange a 10p coin for ten 1p coins.
Now we have 13p – 5p =
$13p - 5p = 8p$

Now: $10p - 30p$

Can we do this calculation?
Exchange a £1 for ten 10p coins and add these into the 10p column.

£1.10 / 110p – 30p =
£1.10 / 110p – 30p = 80p

Then: £3 - £2 =
<table>
<thead>
<tr>
<th>£</th>
<th>10p</th>
<th>1p</th>
</tr>
</thead>
<tbody>
<tr>
<td>£4.23</td>
<td>0.50</td>
<td>0.10</td>
</tr>
<tr>
<td>£2.35</td>
<td>0.50</td>
<td>0.10</td>
</tr>
</tbody>
</table>

£4.23 - £2.35 = £1.88
Websites

Give the Dog a Bone

Eggs to Order

Hickory Dickory clock

Crickweb

Gordons

Topmarks
Some ideas

- Talking Real Maths

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Can you work out if any of these children worked out their answers correctly? If they have made mistakes how could you help them?

If I buy one of everything it will cost £7.90.

- £1.90
- £3.36
- £2.64

0.10
1.80
6.00

- £7.90

Toy Shop

If I buy two slinky toys it will cost me £2.18.

- £1.90 + £1.90 = £1.00 + £1.00 = £2.00
- 9 + 9 + 18
- £2.00 + 18 = £2.18

If I buy a robot and pay with £10.00 I will get £8.46 change.
I know this is right because a robot costs £2.64 and I used my number bonds.

There is a half price offer on for this week only. I think that for £5.00 I can buy one of each toy.
Talking Real Maths

There are 36 chocolates in a layer. How many are in the whole box?

\[
\begin{array}{c}
36 \\
\times 4 \\
\hline
1224 \\
\end{array}
\]

One person is correct. Who do you agree with?

\[
\begin{aligned}
(30 \times 4) + (6\times 4) &= 120 + 30 \\
&= 150 \\
\end{aligned}
\]
Pitch and Expectations

Using and applying mathematics

- Solve one-step and two-step problems involving numbers, money or measures, including time; choose and carry out appropriate calculations, using calculator methods where appropriate.

I pay three pounds sixty pence for a rail ticket. How much change should I get from a five pound note?
Y6 optional test 2003 Mental test level 3

There are 219 boys and 187 girls in a school. What is the total number of children?
Y4 Optional test 1999 Paper B level 3

<table>
<thead>
<tr>
<th>Boat Hire</th>
<th>Motor boats</th>
<th>£1.50 for 15 minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rowing boats</td>
<td>£2.50 for 1 hour</td>
</tr>
</tbody>
</table>

How much does it cost to hire a rowing boat for three hours?
Sasha pays £3.00 to hire a motor boat. She goes out at 3:20 pm. By what time must she return?
K82 2001 Paper B level 3

One length of the swimming pool is 25 metres. Jane swims 5 lengths of the pool. How far does Jane swim altogether?
K82 2001 Paper B level 3

<table>
<thead>
<tr>
<th>Code</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA</td>
<td>75p</td>
</tr>
<tr>
<td>BB</td>
<td>£1.15</td>
</tr>
<tr>
<td>CC</td>
<td>£1.99</td>
</tr>
<tr>
<td>DD</td>
<td>£1.70</td>
</tr>
<tr>
<td>EE</td>
<td>£1.55</td>
</tr>
</tbody>
</table>

Tina buys two cards. One card has code AA on it. The other card has code DD on it. How much does Tina pay?
K82 2002 Paper A level 3

A square playground has a perimeter of 100 metres. How long is one of its sides?
K82 1999 Mental test level 3

<table>
<thead>
<tr>
<th>Fruit</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coconuts</td>
<td>75p each</td>
</tr>
<tr>
<td>Bananas</td>
<td>£1.20 for 3</td>
</tr>
</tbody>
</table>

Josh buys one coconut and half a kilogram of bananas. How much does he spend altogether?
K82 2006 Paper B level 3

One shop sells candles. Each candle costs 75p.
K82 2006 Paper A level 3

<table>
<thead>
<tr>
<th>Code</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left</td>
<td>75p each</td>
</tr>
<tr>
<td>Star</td>
<td>65p each</td>
</tr>
<tr>
<td>Stripe</td>
<td>£1.00 each</td>
</tr>
</tbody>
</table>

Sapna buys 4 star candles and 2 stripe candles. How much does she pay altogether?
K82 2006 Paper A level 3

A special offer: Buy 10 candles and get 50p off.

Buy 10 candles in the special offer. How much does he pay for the 10 candles?
K82 2006 Paper A level 3

This table shows the numbers of children who went walking, sailing or climbing at an outdoor centre.

<table>
<thead>
<tr>
<th>Month</th>
<th>Walking</th>
<th>Sailing</th>
<th>Climbing</th>
</tr>
</thead>
<tbody>
<tr>
<td>May</td>
<td>25</td>
<td>15</td>
<td>18</td>
</tr>
<tr>
<td>June</td>
<td>80</td>
<td>42</td>
<td>27</td>
</tr>
<tr>
<td>July</td>
<td>75</td>
<td>50</td>
<td>23</td>
</tr>
</tbody>
</table>

How many children went sailing in May, June and July altogether?

How many more children went walking in June than climbing in June?
K82 2000 Paper A level 3
Target Questions for Shape

These nets will fold to make 3-D shapes. Match each net to the name of its shape.

- square-based pyramid
- triangular prism
- cube
- square
- tetrahedron
- cuboid

I am thinking of a 3-D shape. It has a square base. It has 4 other faces, which are triangles. What is the name of the 3-D shape?

Y5 optional test 1998 Paper B level 3

How many sides has a quadrilateral?
Y3 optional test 2003 Mental test level 4

How many vertices has a cuboid?
Y4 optional test 2003 Mental test level 4

Look at the diagrams showing 3-D shapes.

This table shows information about four solid shapes. Complete the table. One has been done for you.

<table>
<thead>
<tr>
<th>number of flat surfaces</th>
<th>number of curved surfaces</th>
</tr>
</thead>
<tbody>
<tr>
<td>sphere</td>
<td>0</td>
</tr>
<tr>
<td>cone</td>
<td></td>
</tr>
<tr>
<td>cuboid</td>
<td></td>
</tr>
<tr>
<td>cylinder</td>
<td></td>
</tr>
</tbody>
</table>

KS2 2005 Paper B level 3

One of the shapes has one square face and four triangular faces. Write the letter of this shape.

Two of the shapes have six faces. Write the letters of these shapes.

KS3 2005 Paper A level 3
Talking Real Maths

Sorting Shapes

Can you find these shapes and name them?

Some faces are triangles

Shapes with an even number of faces

I think shapes with more faces have more corners?

3 shapes have been sorted incorrectly. Can you find them?

Now that you've found the incorrect shapes, can you tell me something that is the same and something that is different about them?

What do you think?
Reach 12

The first player places the counter on a number and says the number. The next player then slides the counter onto a new number and adds this to the previous numbers and says the total. Play continues in this way.
Card Games

Activities & Games to Support Learning in Mathematics
Entrust Mathematics Team | June 2013

A range of games and activities using:
• Cards          pages 2 - 7
• Cuisenaire Rods pages 8 - 14
• Dice           pages 15 - 21
• Dominoes       pages 22 - 31

Lower Key Stage 2: Multiple of 10

Learning objective: add and subtract numbers mentally

Remove the picture cards.
Deal 4 cards face up. Place the rest of the cards face down. One card at a time is turned over from the deck. This card can be placed below any of the face up cards.

When a column adds up to a multiple of 10, it can be removed. Can you remove all the columns before the deck runs out?
Bingo with a difference

23  42  12  37
33   2   5  40
25   9  47  32
44  23  17  42

23  33  10

Draw a grid 4 by 4
Write in any numbers between 1 - 50 (as above)
Call out any number between 1-10
Players have to find 2 numbers on their board that have the difference called out
Cross the two numbers off and write them below together (This allows the caller to check the answers from the players)
First player to have a line of crossed numbers is the first winner
The game can be continued until a player has crossed off all numbers on the grid

Instructions with numbers on the cards:-
  The difference of the numbers is restricted between 1-5
  Use differences 9,19,29 or 11,21
  Use the context of measures or time